Queen Mary
University of London

BSc/MSci EXAMINATION

PHY-121 Mathematical Techniques I – Part I

Time Allowed: 50 minutes

Date: **Thursday 18th October 2007**

Time: **12:00 noon**

Answer all questions in section A and two questions from section B. An indicative marking scheme is shown in square brackets [ ] after each part of a question.

Useful information: $\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$, $\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$, $\tan(0)=0$

$$R=\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

DO NOT TURN TO THE FIRST PAGE OF THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

Section A : Questions 1 to 4. Answer all questions.

1) Differentiate the following with respect to x:

a) $(3x+1)^n$ **b)** $\sin x \cos x$ **c)** e^{x^2} **d)** $x^2 e^x$

e) $\dfrac{x}{(2x+1)^2}$ **f)** $\ln(x)$ **g)** $\cos^{-1}(x)$. **[7]**

2) Evaluate all the solutions in the interval $0 \le x < 2\pi$ for the following:

i) $x = \arctan(0)$ **ii)** $x = \arccos\left(\dfrac{1}{2}\right)$ **iii)** $x = \arcsin\left(\dfrac{1}{2}\right)$

[3]

3) A curve is defined parametrically by: $x = a\theta + b\theta^3$ and $y = c\sin\theta$.

a) Calculate $\dfrac{\mathrm{dy}}{\mathrm{d\theta}}$.

b) Calculate $\dfrac{\mathrm{dx}}{\mathrm{d\theta}}$, and $\dfrac{\mathrm{d\theta}}{\mathrm{dx}}$.

c) Hence calculate $\dfrac{\mathrm{dy}}{\mathrm{dx}}$ in terms of θ.

d) Calculate $\dfrac{\mathrm{d}\left(\dfrac{\mathrm{dy}}{\mathrm{dx}}\right)}{\mathrm{d\theta}}$ and $\dfrac{\mathrm{d^2y}}{\mathrm{dx^2}}$ in terms of θ. **[5]**

4) Find all the first and second partial derivatives of $z = x^2 \cos y + xy$ with respect to x and y. **[5]**

Section B: Questions 5 to 7. Answer <u>two</u> questions only.

5) **a)** By *<u>implicit</u>* differentiation find $\frac{\mathrm{dy}}{\mathrm{dx}}$ given the function $R^2 = 3x^2 + y^2$, where R is constant. **[5]**

b) Sketch the function $y = \frac{1}{(m - m_0)^2 + \frac{1}{2}}$ where m_0 is a constant, and determine the values of m and y for the maximum. **[5]**

c) The rest mass of a group of particles moving with a velocity close to the speed of light is given by

$$m = \frac{1}{c^2}\left(E^2 - p^2c^2\right)^{\frac{1}{2}}.$$

Where E is the total energy of the particles, p is the total momentum, and c is the speed of light. By suitable partial differentiation derive an expression for the change in the calculated mass δm due to small changes in the measured energy δE and momentum δp of the particles. Hence calculate the fractional change in calculated mass, $\frac{\delta m}{m}$.

[5]

Total For Question 5 **[15]**

6) **a)** Sketch the function $y = \frac{1}{x^2}$, and prove that it has no stationary points. **[5]**

b) Show that the function: $y = e^{3x}$ is a solution to the equation:

$$\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2} = Ay,$$

and find the value of the parameter A. **[5]**

c) Calculate $d\theta/dy$ by *<u>implicit</u>* differentiation for the function

$$y = (a\theta + by)\cos^2\theta,$$

where a and b are constants. **[5]**

Total For Question 6 **[15]**

7) **a)** Calculate the radius of curvature R of the function $y = 4x^2 - 2x$ at $x = 0$. **[5]**

b) Find $\frac{\mathrm{dy}}{\mathrm{dx}}$ in terms of t where $y = A\cos\pi t$ and $x = B\tan\pi t$. **[5]**

c) Find the stationary points of the function $y = x^3 + 2x^2 + x$, and indicate any maximum, minimum or point of inflection.

[5]

Total For Question 7 **[15]**

End of Examination Paper **Dr A. Bevan**