COSMOLOGY I & II Help sheet 2010

1 eV $=11600$ K $= 1.60 \times 10^{-19}$J $= 5.07 \times 10^{6}\mathrm{m}^{-1} = 1.52 \times 10^{15}\mathrm{s}^{-1} = 1.78 \times 10^{-36}$kg

$c = 1 = 3.00 \times 10^{8}$m/s

$\hbar = 1 = 197$ MeVfm

1 pc $= 3.09 \times 10^{16}$m

1 a $= 3.156 \times 10^{7}$s

$h \equiv H_0/(100\mathrm{km/s/Mpc})$

$(100\mathrm{km/s/Mpc})^{-1} = 9.78 \times 10^{9}\mathrm{a} = 2998$ Mpc

$T_0 = 2.725$ K $= 2.348 \times 10^{-4}$ eV

$T_{\nu 0} = (4/11)^{1/3} T_0$

$z_{\mathrm{dec}} = 1100$

$m_e = 0.511$ MeV

$m_N = 938$ MeV

$m_p + m_e - m_H = 13.6$ eV

$\zeta(3) = 1.20206$

$m_{\mathrm{Pl}} \equiv G^{-1/2} = 1.22 \times 10^{22}$ MeV

$M_{\mathrm{Pl}} \equiv (8\pi G)^{-1/2} = 2.436 \times 10^{21}$ MeV

$g_n = g_p = g_e = 2, \qquad g_H = 4$

$Q = m_n - m_p = 1.293$ MeV

$g_*(T \ll m_e) = 3.363$

$g_{*S}(T \ll m_e) = 3.909$

$g_*(1\mathrm{MeV}) = g_{*S}(1\mathrm{MeV}) = 10.75$

$\tau_n = t_{1/2}/\ln 2 = 886\mathrm{s}$

$n + \nu_e \leftrightarrow p + e^-$

$\mu_e \ll T \qquad (\text{when } T > 30\mathrm{keV})$

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

$$n_i = \left\{ \begin{array}{c} 1 \\ 3/4 \end{array} \right\} \frac{\zeta(3)}{\pi^2} g_i T^3, \quad (T \gg m_i)$$

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-\frac{m_i - \mu_i}{T}}, \quad (T \ll m_i)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\rho = \frac{\pi^2}{30} g_* T^4$$

$$s = \frac{2\pi^2}{45} g_{*S} T^3$$

$${g_*}^{1/2} t T^2 = 0.301 m_{\mathrm{Pl}}$$

$$\rho = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\nabla\varphi^2 + V(\varphi)$$

$$\ddot{\varphi} - \frac{1}{R^2}\nabla^2\varphi + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$n - \bar{n} = \frac{gT^3}{6\pi^2}\left[\pi^2\left(\frac{\mu}{T}\right) + \left(\frac{\mu}{T}\right)^3\right], \quad (T \gg m)$$

$$n - \bar{n} = 2g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \sinh\frac{\mu}{T}, \quad (T \ll m)$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} + \frac{1}{\rho}\nabla p + \nabla\tilde{\Phi} = 0$$

$$\nabla^2\tilde{\Phi} = 4\pi G\rho$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \mathrm{arsinh(x)}$$

$$\mathcal{R} = -H\frac{\delta\varphi}{\dot{\varphi}}$$

$$\mathcal{P}_g(k) \equiv \left(\frac{L}{2\pi}\right)^3 4\pi k^3 \langle |g_{\mathbf{k}}|^2 \rangle$$

$$\left(\frac{2\pi}{L}\right)^3 \sum_{\mathbf{k}} \to \int d^3k$$

$$\mathcal{P}_\varphi(k) = \left(\frac{H}{2\pi}\right)^2_{|aH=a_0k}$$

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\varphi}}\right)^2 \left(\frac{H}{2\pi}\right)^2_{|aH=a_0k}$$

$$n(k) - 1 \equiv \frac{d\ln\mathcal{P}_{\mathcal{R}}}{d\ln k}$$

$$\left(\frac{\delta T}{T}\right)_{\mathrm{obs}} = \frac{1}{4}\delta_\gamma^N - \mathbf{v}^N\cdot\hat{\mathbf{n}} + \Phi(t_{\mathrm{dec}}, \mathbf{x}_{\mathrm{ls}}) + 2\int\dot{\Phi}dt$$

$$e^{i\mathbf{k}\cdot\mathbf{x}} = 4\pi\sum_{lm} i^l j_l(kx) Y_{lm}(\hat{\mathbf{x}}) Y^*_{lm}(\hat{\mathbf{k}})$$

$$\Phi_{\mathbf{k}} \equiv -\frac{3}{2}\left(\frac{aH}{a_0k}\right)^2 \delta_{\mathbf{k}}$$

$$\Phi_{\mathbf{k}} = -\frac{3+3w}{5+3w}\mathcal{R}_{\mathbf{k}}, \quad (w \equiv p/\rho)$$

$$\int d\Omega\, Y^*_{lm}(\theta,\phi) Y_{l'm'}(\theta,\phi) = \delta_{ll'}\delta_{mm'}$$

$$\sum_m \left|Y_{lm}(\theta,\phi)\right|^2 = \frac{2l+1}{4\pi}$$

$$\int_0^\infty \frac{dz}{z} j_l(z)^2 = \frac{1}{2l(l+1)}$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

The particles in the standard model and $g_*(T)$
Particle Data Group, 2010

Quarks	t	173.0 ± 1.6 GeV	$\bar{t}$	spin=$\frac{1}{2}$	$g_* = \frac{7}{8} \cdot 2 \cdot 2 \cdot 3 = 10.5$	
	b	4.19 ± 0.18 GeV	$\bar{b}$	3 colours		
	c	1.27 ± 0.09 GeV	$\bar{c}$			
	s	101 ± 29 MeV	$\bar{s}$			
	d	4.1–5.8 MeV	$\bar{d}$			
	u	1.7–3.3 MeV	$\bar{u}$			
						63
Gluons	8 massless bosons			spin=1	$g_* = 2$	16
Leptons	τ^-	1776.82 ± 0.16 MeV	τ^+	spin=$\frac{1}{2}$	$g_* = \frac{7}{8} \cdot 2 \cdot 2 = 3.5$	
	μ^-	105.658 MeV	μ^+			
	e^-	510.999 keV	e^+			
						10.5
	ν_τ	< 2eV	$\bar{\nu}_\tau$	spin=$\frac{1}{2}$	$g_* = \frac{7}{8} \cdot 2 = 1.75$	
	ν_μ	< 2eV	$\bar{\nu}_\mu$			
	ν_e	< 2 eV	$\bar{\nu}_e$			
						5.25
Electroweak gauge bosons	$W^\pm$	80.399 ± 0.023 GeV		spin=1	$g_* = 2 \cdot 3 \to 3 \cdot 3$	
	Z^0	91.1876 ± 0.0021 GeV				
	γ	0 $(< 1 \times 10^{-18}$ eV)			$g_* = 2$	
						$8 \to 11$
Higgs boson (SM)	H^0	> 114.4 GeV		spin=0	$g_* = 4 \to 1$	$4 \to 1$
						106.75

History of $g_*(T)$

$T \sim 200$ GeV	all present	106.75		
$T \sim 100$ GeV	EW transition	106.75 (no effect)		
$T < 100$ GeV	top-annihilation	96.25		
$T < 80$ GeV	$W^\pm$, Z^0, H^0	86.25		
$T < 4$ GeV	bottom	75.75		
$T < 1$ GeV	charm, τ^-	61.75		
$T \sim 150$ MeV	QCD transition	17.25	(u,d,g$\to \pi^{\pm,0}$,	$37 \to 3$)
$T < 100$ MeV	$\pi^\pm$, π^0, μ^-	10.75	$e^\pm$, ν, $\bar{\nu}$, γ left	
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$	