October 2, 2008

Mathematical Techniques: Lecture 1 Revision Notes

Dr A. J. Bevan,

These notes contain the core of the information conveyed in the lectures. They are not a substitute for attending the lectures and none of the examples covered are reproduced here. Worked examples of the techniques described in this note can be found in the tutorial question/solution material provided on the course web site.

1 Rates of change

The change in y with respect to x between these two points on the curve is given by the ratio

$$\begin{aligned}\frac{\delta y}{\delta x} &= \frac{y_2 - y_1}{x_2 - x_1}, \\ &= \frac{y(x+\delta x) - y(x)}{\delta x}. \end{aligned} \tag{1.1}$$

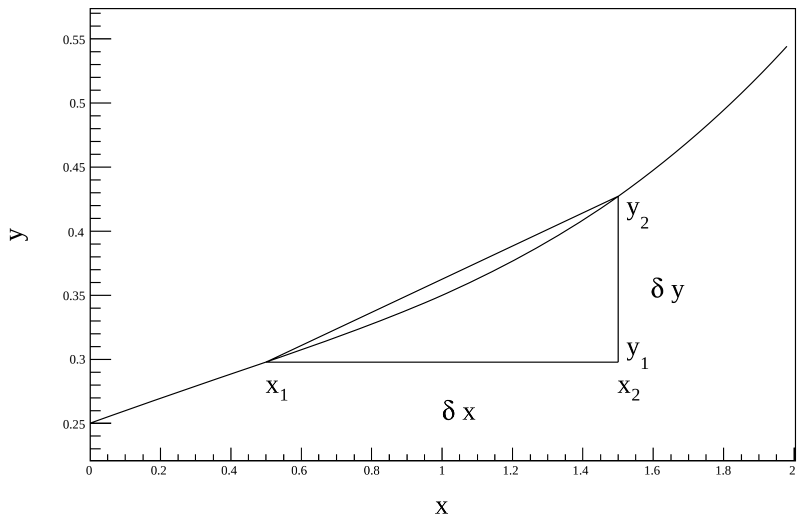

Figure 1: The function $y = f(x)$.

If we take the limit as δx tends to zero we obtain

$$\begin{aligned}\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right) &= \frac{dy}{dx}, \\ &= \lim_{\delta x \to 0}\left(\frac{y(x+\delta x) - y(x)}{\delta x}\right). \end{aligned} \tag{1.2}$$

which is exact. In general the derivative obtained will be a function of x and can itself be differentiated in order to obtain higher order derivatives.

1.1 Standard Derivatives

It is tedious to work out the derivatives of common functions using Eq. (1.2) each time, and in practice we rely on tables or books of 'standard derivatives'. Table 1 lists a number of useful results.

Table 1: Table of standard derivatives.

$y = f(x)$	$\frac{dy}{dx}$
x^n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec} x$	$-\text{cosec} x \cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$1/\cosh^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$

2 Chain Rule

Consider the function $y = f(u)$, where $u = g(x)$. Here y is a function of u, which itself is a function of some other variable x. The chain rule states that for such a function

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}. \tag{2.1}$$