October 2, 2008

Mathematical Techniques: Lecture 2 Revision Notes

Dr A. J. Bevan,

These notes contain the core of the information conveyed in the lectures. They are not a substitute for attending the lectures and none of the examples covered are reproduced here. Worked examples of the techniques described in this note can be found in the tutorial question/solution material provided on the course web site.

3 Differentiation of a Products

The rule for the differentiation of the product of two functions $u(x)$ and $v(x)$ is

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}, \tag{3.1}$$

where it is assumed that both u and v are differentiable with respect to x.

4 Differentiation of Quotients

The rule for the differentiation of the ratio of two functions $u(x)$ and $v(x)$ is

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}, \tag{4.1}$$

where it is assumed that both u and v are differentiable with respect to x.

5 Logarithmic Differentiation

If one has a more complicated function, such as

$$y = \frac{u^n(x)v^m(x)}{h^p(x)}, \tag{5.1}$$

while it is possible to recursively use the quotient and product rules to compute the derivative, it is possible, and often convenient to use natural logarithms to simplify the problem at hand. Taking the log of both sides of Eq. (5.2) one obtains

$$\ln y = n \ln u(x) + m \ln v(x) - p \ln h(x), \tag{5.2}$$

which can be differentiated, noting that y is a function of x so that

$$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx}, \tag{5.3}$$

thus

$$\frac{1}{y}\frac{dy}{dx} = \frac{n}{u}\frac{du}{dx} + \frac{m}{v}\frac{dv}{dx} - \frac{p}{h}\frac{dh}{dx}, \tag{5.4}$$

which can be used to determine the derivative as a function of x given the form of u, v and h.

6 Implicit Differentiation

It is possible to differentiate functions that are implicitly relating y to x using the chain rule (as with the previous example). For example, consider

$$x^2 + y^2 = R^2, \tag{6.1}$$

which describes a circle. The derivative of this equation is

$$2x + 2y\frac{dy}{dx} = 0, \tag{6.2}$$

so

$$\frac{dy}{dx} = -\frac{x}{y}. \tag{6.3}$$

7 Parametric Functions

If we have a function defined by

$$y = h(\theta) \text{ and } x = g(\theta), \tag{7.1}$$

then we can compute the derivative of y with respect to x by using the chain rule, and noting that

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}. \tag{7.2}$$

assuming that it is possible do differentiate both h and g with respect to θ.