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Mathematical Techniques: Lecture 3 Revision Notes

Dr A. J. Bevan,

These notes contain the core of the information conveyed in the lectures. They are not a substitute for attending the lectures and none of the examples covered are reproduced here. Worked examples of the techniques described in this note can be found in the tutorial question/solution material provided on the course web site.

8 Tangents and normals to a curve

Consider a straight line where

$$y = mx + C, \tag{8.1}$$

and

$$m = \frac{dy}{dx}, \tag{8.2}$$

which is the slope of the line. Given a point on the line (x_1, y_1) it is possible to compute the equation of the line by noting that

$$y - y_1 = m(x - x_1), \tag{8.3}$$

$$y = mx - mx_1 + y_1, \tag{8.4}$$

which is the equation of the tangent to a curve with gradient m at the point (x_1, y_1).

The equation of the normal to a curve at (x_1, y_1) can be obtained by recalling that the gradient of the normal to a curve is $-1/m$. So the equation of the normal to a curve is just

$$y = -\frac{1}{m}x + \frac{1}{m}x_1 + y_1. \tag{8.5}$$

9 Differentiating inverse trigonometric functions

The table of standard derivatives (Table 1) lists the derivatives of the trigonometric functions. These can be used in order to derive the rules for differentiating inverse trigonometric functions. If we consider the equation

$$y = \arcsin(x),$$

we are able to take the sine of both sides to obtain

$$\sin(y) = x.$$

We can now differentiate x with respect to y which yields

$$\begin{aligned}\frac{dx}{dy} &= \cos(y), \\ &= \sqrt{1-x^2},\end{aligned}$$

using $\sin^2(y) + \cos^2(y) = 1$. We can now readily obtain the derivative of $\arcsin(x)$ which is

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

It is possible to show that the derivative of $y = \arccos(x)$ is

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

and similar steps can be taken in order to show that for $y = \arctan(x)$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$